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A simple sum rule for the thermal gluon spectral function and applications

Patrick Aurenche
Laboratoire d'Annecy-le-Vieux de Physique Théorique
Chemin de Bellevue, B.P. 110
74941 Annecy-le-Vieux Cedex, France
E-mail: aurenche@lapp.in2p3.fr

François Gelis
Laboratoire de Physique Théorique, Bât. 210, Université Paris XI
91405 Orsay Cedex, France
E-mail: gelis@th.u-psud.fr

Haitham Zaraket
Physics Department and Winnipeg Institute for Theoretical Physics
University of Winnipeg
Winnipeg, Manitoba R3B 2E9, Canada
E-mail: zaraket@theory.uwinnipeg.ca

Abstract: In this paper, we derive a simple sum rule satisfied by the gluon spectral function at finite temperature. This sum rule is useful in order to calculate exactly some integrals that appear frequently in the photon or dilepton production rate by a quark gluon plasma. Using this sum rule, we rederive simply some known results and obtain some new results that would be extremely difficult to justify otherwise. In particular, we explain how this result can be used to calculate the photon rate in a simple quasi-particle model which correctly reproduces the thermodynamic properties of a quark-gluon plasma. We also derive an exact expression for the collision integral that appears in the calculation of the Landau-Pomeranchuk-Migdal effect and propose a method to solve the resulting differential equation.

Keywords: Thermal Field Theory, QCD, Sum Rules.

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1. Introduction

Photon production is considered to be a very interesting signal of the formation of a quark gluon plasma in heavy ion collisions [1]–[7]. Indeed, because of their very weak coupling to matter, photons (and more generally any electromagnetic probe) have a large mean free path which enables them to escape without reinteractions from a medium the size of which is at best a few tens of fermis.

On the theoretical side, the calculation of the photon and dilepton rate is performed under the hypothesis of local equilibrium, i.e. one calculates a rate (number of photons produced per unit time and per unit volume) using thermal field theory in equilibrium,

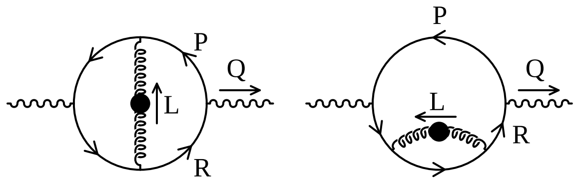

Figure 1: The two-loop diagrams contributing to photon and dilepton production.

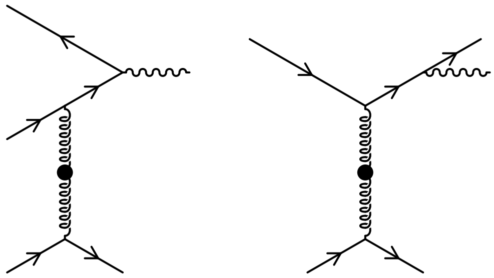

Figure 2: Important processes contained in the above 2-loop diagrams.

and then plugs this rate into some hydrodynamical model [1], [4]–[7] which describes the system by dividing it in cells where a local equilibrium is assumed and by assigning a local temperature and fluid 4-velocity to each cell. Such a description is consistent as long as the photon formation time is small compared to the typical size of the cells in which an approximate equilibrium is realized [8, 9].

Thermal field theory calculations of photon and dilepton production rates have been performed a long time ago [10]–[13], and have been reassessed under the new light shaded by the concept of hard thermal loops (HTL — [14]–[18]) [19]–[24]. In this context, it has been found that some $2 \to 3$ and $3 \to 2$ processes which appear only in 2-loop diagrams [25]–[28] (like bremsstrahlung, as well as the process $q\bar{q}\{q,g\} \to \gamma\{q,g\}$ which can be deduced from bremsstrahlung by crossing symmetry - see figures 1 and 2) are also important.

These processes are in fact enhanced by a strong sensitivity to the forward emission of the photon, due to a collinear singularity regularized by a thermal mass of order gT where g is the gauge coupling. In particular, the process on the left of figure 2 has been found to enhance considerably the rate of hard photons [27]. This collinear enhancement mechanism has also been shown to play a role in multi-loop diagrams belonging to the class of ladder corrections and self-energy-corrections [8], [29]–[33], and a resummation of this family of diagrams has been carried out in [32, 33]. The effect of this resummation, also known as Landau-Pomeranchuk-Migdal (LPM) effect [34, 35, 36], leads to a small reduction (by about 25% for photons in the range interesting for phenomenology) of the photon rate.

It is expected that the strength of the LPM corrections is comparable for the production of photon with a small invariant mass, and smaller for photons with an invariant mass large compared to M_∞^2 (this statement is however not true for a photon mass close to the threshold $2M_\infty$ for $q\bar{q}$ annihilation, because there are large perturbative corrections in this region). As a first attempt to estimate the contribution to dilepton production of the off-shell annihilation process of figure 2, we have extended recently our 2-loop calculation of [27] to the case where the photon mass cannot be neglected any longer, leaving the problem of resumming the LPM corrections for a later work. In this paper, we obtained a simple generalization to the case of massive photons of the formula known for the imaginary part of the real photon polarization tensor. This formula reads:

$$\begin{aligned}\mathrm{Im}\,\Pi_{R\,\mu}^{\ \mu}(Q) \approx & -\frac{e^2 g^2 N_c C_F}{2\pi^4}\frac{T}{q_0^2}\int_{-\infty}^{+\infty} dp_0\, [n_F(r_0)-n_F(p_0)] \times \\ & \times \left\{ \left(p_0^2+r_0^2\right)(J_T - J_L) + 2\frac{Q^2 p_0 r_0 + M_\infty^2\left(p_0^2+r_0^2\right)}{M_{\rm eff}^2}(K_T - K_L)\right\}. \end{aligned} \tag{1.1}$$

with $r_0 \equiv p_0 + q_0$ and[1]

$$\begin{aligned} J_{T,L} &\equiv M_{\rm eff}^2 \int_0^1 \frac{dx}{x}\,\mathrm{Im}\,\Pi_{T,L}(x)\int_0^{+\infty} dw \frac{\sqrt{w/(w+4)}\tanh^{-1}\sqrt{w/(w+4)}}{(M_{\rm eff}^2 w + \mathrm{Re}\,\Pi_{T,L}(x))^2 + (\mathrm{Im}\,\Pi_{T,L}(x))^2}, \\ K_{T,L} &\equiv M_{\rm eff}^2 \int_0^1 \frac{dx}{x}\,\mathrm{Im}\,\Pi_{T,L}(x)\int_0^{+\infty} \frac{dw}{w} \frac{\sqrt{w/(w+4)}\tanh^{-1}\sqrt{w/(w+4)} - w/4}{(M_{\rm eff}^2 w + \mathrm{Re}\,\Pi_{T,L}(x))^2 + (\mathrm{Im}\,\Pi_{T,L}(x))^2}, \end{aligned} \tag{1.2}$$

where the $\Pi_{T,L}$ are the transverse and longitudinal self-energies resummed on the gluon propagator, and where we denote

$$M_{\rm eff}^2 \equiv M_\infty^2 + \frac{Q^2}{q_0^2} p_0 r_0\,, \tag{1.3}$$

with M_∞ the thermal mass of a hard quark ($M_\infty^2 = g^2 C_F T^2/4$). The functions $\Pi_{T,L}$ are the usual transverse and longitudinal HTL gluon self-energies:

$$\begin{aligned} \Pi_T(L) &= 3m_{\rm g}^2\left[\frac{x^2}{2} + \frac{x(1-x^2)}{4}\ln\left(\frac{x+1}{x-1}\right)\right] \\ \Pi_L(L) &= 3m_{\rm g}^2\left[(1-x^2) - \frac{x(1-x^2)}{2}\ln\left(\frac{x+1}{x-1}\right)\right], \end{aligned} \tag{1.4}$$

where we denote $x \equiv l_0/l$ and where $m_{\rm g}^2 \equiv g^2 T^2 [N_c + N_F/2]/9$ is the gluon thermal mass in a SU(N_c) gauge theory with N_F flavors.

Therefore, in addition to the function $J_{T,L}$ already introduced in the case of quasi-real photons,[2] we need two new functions $K_{T,L}$ for the term proportional to the photon invariant mass squared Q^2. All are dimensionless functions of the ratio of $M_{\rm eff}$ to the plasmon mass $m_{\rm g}$ which appears as a prefactor in the self-energies $\Pi_{T,L}$. Up to now, the $J_{T,L}$ and $K_{T,L}$ have only been evaluated numerically ([26, 27], with a mistake corrected by [38, 39, 40]),

[1] Let us recall here that $M_{\rm eff}^2$ can become negative if $Q^2 > 4M_\infty^2$. Those definitions are only appropriate for $M_{\rm eff}^2 > 0$, because they have been obtained by rescaling the transverse momentum $l_\perp$ of the gluon by writing $l_\perp^2 \equiv M_{\rm eff}^2 w$. However, it was argued in [37] that the correct result for $M_{\rm eff}^2 < 0$ can be obtained as the real part of the analytic continuation of the result for $M_{\rm eff}^2 > 0$. Most of this paper deals with the case of a positive $M_{\rm eff}^2$.

[2] The term in $M_\infty^2 K_{T,L}$ was forgotten in [26]. It comes from the HTL correction to the $\gamma q\bar{q}$ vertex. This vertex correction was also neglected in [32, 33] without any damage to this approach since it affects only the component Π_{zz} of the polarization tensor, while only the transverse components are calculated in these papers (see [37] for more details on this issue).

which is sufficient for the case of real photons since in this case they are fixed numbers that depend only on the number of colors and flavors but not on kinematical parameters ($M^2_{\rm eff} = M^2_\infty$). However, the cost of this procedure increases significantly in the case of virtual photons since the value of $M^2_{\rm eff}$ depends on the invariant mass Q, energy q_0 and quark energy p_0. In addition, obtaining asymptotic limits is far from trivial at this point.[3]

The aim of the present paper is to derive some analytical results regarding those functions. We first show how the integral over the variable x can be performed exactly in the functions $J_{T,L}$ and $K_{T,L}$ by means of a simple sum-rule (section 2). This leads to either a very simple integral representation of these functions or even to closed formulas in terms of dilogarithms (section 3). Thanks to these results, we can easily study the asymptotic properties of the functions $J_{T,L}$ and $K_{T,L}$. Non trivial asymptotic expansions are obtained, which would have been very difficult to obtain otherwise (section 4). In section 5, we show that the above analytic results can also give some insight on the fact that the processes of figure 2 depend only on parameters like the gluon screening masses and the hard quark thermal mass, in a generic model where the quark gluon plasma is described as a gas of quasi-particles. Finally, we show that one can also calculate analytically the collision integral that appears in the resummation of ladder diagrams [32, 33] (section 6). In appendix A, we derive the asymptotic behavior of a function introduced at an intermediate stage. In appendix B, we prove analytically an anecdotical property which was first noticed numerically [41]: the integrals J_T and J_L are exactly opposite if $M_\infty = m_{\rm g}$ and $Q^2 = 0$.

2. Derivation of the sum-rule

We want to calculate the integral:

$$f(z) \equiv \int_0^1 \frac{dx}{x} \frac{2\,{\rm Im}\,\Pi(x)}{(z + {\rm Re}\,\Pi(x))^2 + ({\rm Im}\,\Pi(x))^2}\,, \tag{2.1}$$

for a positive z, where $\Pi(x)$ is some self-energy depending only on $x \equiv k_0/k$ as is the case for instance with the HTL gluonic self-energy. The factor $1/x$ comes from a Bose-Einstein factor in the soft approximation $dk_0 n_{B}(k_0) \approx T dk_0/k_0 = T dx/x$. The first step in this calculation is to rewrite it as

$$f(z) = \int_0^1 \frac{dx}{x}(1-x^2)\frac{2\,{\rm Im}\,\overline{\Pi}(x)}{(z(x^2-1) - {\rm Re}\,\overline{\Pi}(x))^2 + ({\rm Im}\,\overline{\Pi}(x))^2}\,, \tag{2.2}$$

where we define $\overline{\Pi}(x) \equiv (1-x^2)\Pi(x)$. Interpreting now z ($z > 0$) as the square of a three-momentum k and x as the ratio $x = k_0/k$, we have

$$\begin{aligned}\frac{-2\,{\rm Im}\,\overline{\Pi}(x)}{(z(x^2-1) - {\rm Re}\,\overline{\Pi}(x))^2 + ({\rm Im}\,\overline{\Pi}(x))^2} &= 2\,{\rm Re}\,\frac{i}{k_0^2 - k^2 - \overline{\Pi}(k_0,k) + ik_0\epsilon} \\ &\equiv \overline{\rho}(k_0,k)\,.\end{aligned} \tag{2.3}$$

[3] Some very partial asymptotic results have been obtained in [26] for $J_{T,L}$.

Note that this function is nothing but the spectral function $\overline{\rho}(k_0, k)$ associated with the "propagator" appearing in the right hand side. This is what enables us to relate the integral

$$f(z) = -\int_0^1 \frac{dx}{x}(1-x^2)\overline{\rho}(\sqrt{z}x, \sqrt{z}) \tag{2.4}$$

to the spectral representation of this propagator. Indeed, it is known that the resummed propagator $i/(K^2 - \overline{\Pi}(K))$ is related to its spectral function $\overline{\rho}(k_0, k)$ via the following spectral representation [42, 43]:

$$\frac{i}{k_0^2 - k^2 - \overline{\Pi}(k_0/k) + ik_0\epsilon} = \int_0^{+\infty} \frac{dE}{\pi} E\overline{\rho}(E, k)\frac{i}{k_0^2 - E^2 + ik_0\epsilon}\,. \tag{2.5}$$

Taking the real part of this identity,[4] one recovers the definition eq. (2.3) of the spectral function. Taking its imaginary part and denoting $E \equiv kx$ and $k_0 = ky$, one obtains the following non-trivial integral:

$$\int_0^{+\infty} \frac{dx}{\pi} x\overline{\rho}(kx, k)\frac{1}{y^2 - x^2} = \frac{k^2(y^2-1) - \mathrm{Re}\,\overline{\Pi}(y)}{(k^2(y^2-1) - \mathrm{Re}\,\overline{\Pi}(y))^2 + (\mathrm{Im}\,\overline{\Pi}(y))^2}\,. \tag{2.6}$$

Taking the limit $y \to \infty$, we find

$$\int_0^{+\infty} \frac{dx}{\pi} x\overline{\rho}(kx, k) = \frac{1}{k^2 - \lim_{y\to\infty} \mathrm{Re}\,\overline{\Pi}(y)/y^2} = \frac{1}{k^2 + \mathrm{Re}\,\Pi(\infty)}\,. \tag{2.7}$$

Having in mind that $\Pi(y)$ is a gluonic self-energy obtained from the hard thermal loop approximation, its imaginary part vanishes if $y \geq 1$ and therefore does not contribute in the limit $y \to \infty$. Alternatively, taking the limit $y \to 0$ and assuming similarly that $\mathrm{Im}\,\Pi(y = 0) = 0$, we obtain:

$$\int_0^{+\infty} \frac{dx}{\pi}\frac{1}{x}\overline{\rho}(kx, k) = \frac{1}{k^2 + \mathrm{Re}\,\Pi(0)}\,. \tag{2.8}$$

We can combine these two relations into

$$\int_0^{+\infty} \frac{dx}{x}(1-x^2)\overline{\rho}(kx, k) = \pi\left[\frac{1}{k^2 + \mathrm{Re}\,\Pi(0)} - \frac{1}{k^2 + \mathrm{Re}\,\Pi(\infty)}\right]. \tag{2.9}$$

In order to obtain from there the function $f(z)$, we need to subtract the contribution coming from x between 1 and $+\infty$. Fortunately, since $\mathrm{Im}\,\Pi(x) = 0$ for $x \geq 1$, the contribution in this range comes only from the poles of the propagator $i/(K^2 - \overline{\Pi}(K))$, via the formula[5]

$$\int_1^{+\infty} \frac{dx}{x}(1-x^2)\overline{\rho}(kx, k) = \pi \sum_{\text{poles } x_i} \frac{Z(x_i)}{k^2}\frac{1-x_i^2}{x_i^2}\,, \tag{2.10}$$

where $Z(x_i)$ is the residue of the propagator at the corresponding pole. For the above propagator, the equation that determines the poles is

$$k^2(x^2-1) = \mathrm{Re}\,\overline{\Pi}(x) = (1-x^2)\,\mathrm{Re}\,\Pi(x)\,. \tag{2.11}$$

[4]Note that the spectral function $\overline{\rho}(k_0, k)$ is by definition a real function.

[5]If $\mathrm{Im}\,\overline{\Pi}(x) = 0$, then $\overline{\rho}(kx, k) = 2\pi\delta(k^2(x^2-1) - \overline{\Pi}(x))$.

This "dispersion equation" has a trivial solution $x = 1$, which does not contribute when plugged in eq. (2.10) because the other factors in the integrand vanish if $x = 1$. Any non trivial pole would be a solution of the equation

$$k^2 + \mathrm{Re}\,\Pi(x) = 0\,, \tag{2.12}$$

but under the reasonable assumption that the resummation of the self-energy $\Pi(x)$ leads to well behaved quasi-particles (i.e. that the equation $k_0^2 - k^2 = \Pi(k_0/k)$ has a solution for every value of k_0/k larger than 1), we have $\mathrm{Re}\,\Pi(x) \geq 0$ for $x > 1$ and therefore there are no additional poles. As a consequence, the integral of eq. (2.9) does not receive any contribution from the range $x \in [1, +\infty]$, and we can write directly a closed expression for the function $f(z)$:

$$f(z) = \int_0^1 \frac{dx}{x} \frac{2\,\mathrm{Im}\,\Pi(x)}{(z + \mathrm{Re}\,\Pi(x))^2 + (\mathrm{Im}\,\Pi(x))^2} = \pi \left[\frac{1}{z + \mathrm{Re}\,\Pi(\infty)} - \frac{1}{z + \mathrm{Re}\,\Pi(0)} \right]. \tag{2.13}$$

This is the basic sum-rule from which we are going to derive some results regarding photon production by a quark-gluon plasma. The validity of this result can also be checked numerically.

It may be useful to recall here the assumptions we have made about the self-energy Π in order to obtain this result:

1. Π depends only on $x \equiv k_0/k$
2. $\mathrm{Im}\,\Pi(x = 0) = 0$
3. $\mathrm{Im}\,\Pi(x) = 0$ if $x \geq 1$
4. $\mathrm{Re}\,\Pi(x) \geq 0$ if $x \geq 1$.

The condition (2) is always true and condition (4) is not very restrictive, but the conditions (1) and (3) are a priori only valid at leading order in g.

In [26], a formula for the photon rate based on sum rules was also presented. However, in this paper, the use of sum rules led only to a very complicated result (involving explicitly the gluon dispersion equations as well as the residues of the gluon poles) that was not useful for any practical purpose. By comparing the two methods, we can trace the simplification achieved in the present paper to a different choice for the integration variables. Indeed, in [26] the sum rules were applied to an integral over the variables $x \equiv l_0/l$, $l = |\boldsymbol{l}|$ (where L is the momentum of the exchanged gluon), while in the present approach, we take as independent integration variables x and $w \equiv -L^2/M_{\mathrm{eff}}^2 = l^2(1 - x^2)/M_{\mathrm{eff}}^2$. It appears that trading l in favor of w before using sum rules to perform the integral over x leads to a dramatic simplification of the result, because some non trivial parts of the x dependence get absorbed in the new variable w. There are in fact many sum rules satisfied by the HTL spectral functions. The interested reader may find other examples in [44, 45, 46, 47].

3. Expression of $J_{T,L}$ and $K_{T,L}$

Thanks to the formula derived in the previous section, one can first simplify the functions $J_{T,L}$ and $K_{T,L}$ so that we have only one-dimensional integrals over the variable w:

$$\begin{aligned}
J_{T,L} &= \frac{\pi}{2}\int_0^{+\infty} dw \sqrt{\frac{w}{w+4}} \tanh^{-1}\sqrt{\frac{w}{w+4}} \times \\
&\qquad \times \left[\frac{1}{w+\frac{\mathrm{Re}\,\Pi_{T,L}(\infty)}{M_{\mathrm{eff}}^2}} - \frac{1}{w+\frac{\mathrm{Re}\,\Pi_{T,L}(0)}{M_{\mathrm{eff}}^2}}\right], \\
K_{T,L} &= \frac{\pi}{2}\int_0^{+\infty} \frac{dw}{w}\left[\sqrt{\frac{w}{w+4}} \tanh^{-1}\sqrt{\frac{w}{w+4}} - \frac{w}{4}\right] \times \\
&\qquad \times \left[\frac{1}{w+\frac{\mathrm{Re}\,\Pi_{T,L}(\infty)}{M_{\mathrm{eff}}^2}} - \frac{1}{w+\frac{\mathrm{Re}\,\Pi_{T,L}(0)}{M_{\mathrm{eff}}^2}}\right].
\end{aligned} \tag{3.1}$$

At this point, using the change of variable $u \equiv \sqrt{w/(w+4)}$, we can write in a simpler way the following elementary integrals:

$$\int_0^{+\infty} dw \sqrt{\frac{w}{w+4}} \tanh^{-1}\sqrt{\frac{w}{w+4}}\left[\frac{1}{w} - \frac{1}{w+a}\right] = 2F\left(\frac{4}{a}\right), \tag{3.2}$$

$$\int_0^{+\infty} \frac{dw}{w}\left[\sqrt{\frac{w}{w+4}} \tanh^{-1}\sqrt{\frac{w}{w+4}} - \frac{w}{4}\right]\left[\frac{1}{w} - \frac{1}{w+a}\right] = \frac{1}{4}\ln\left(\frac{1}{a}\right) + \frac{1}{2} - \frac{2}{a}F\left(\frac{4}{a}\right),$$

where we define the function

$$F(x) \equiv \int_0^1 du \frac{\tanh^{-1}(u)}{(x-1)u^2+1}. \tag{3.3}$$

Recalling now the following properties of the HTL self-energy of the gluon [14]

$$\begin{aligned}
\mathrm{Re}\,\Pi_{T,L}(\infty) &= m_{\mathrm{g}}^2 \\
\mathrm{Re}\,\Pi_{T}(0) &= 0 \\
\mathrm{Re}\,\Pi_{L}(0) &= 3m_{\mathrm{g}}^2
\end{aligned} \tag{3.4}$$

where m_{g} is the plasmon mass, we easily obtain the following expressions in terms of the function $F(x)$:

$$\begin{aligned}
J_T &= -\pi F\left(\frac{4M_{\mathrm{eff}}^2}{m_{\mathrm{g}}^2}\right), \\
J_L &= \pi\left[F\left(\frac{4M_{\mathrm{eff}}^2}{3m_{\mathrm{g}}^2}\right) - F\left(\frac{4M_{\mathrm{eff}}^2}{m_{\mathrm{g}}^2}\right)\right], \\
K_T &= \pi\left[\frac{M_{\mathrm{eff}}^2}{m_{\mathrm{g}}^2}F\left(\frac{4M_{\mathrm{eff}}^2}{m_{\mathrm{g}}^2}\right) - \frac{1}{4} - \frac{1}{8}\ln\left(\frac{M_{\mathrm{eff}}^2}{m_{\mathrm{g}}^2}\right)\right], \\
K_L &= \pi\left[-\frac{1}{8}\ln(3) + \frac{M_{\mathrm{eff}}^2}{m_{\mathrm{g}}^2}\left(F\left(\frac{4M_{\mathrm{eff}}^2}{m_{\mathrm{g}}^2}\right) - \frac{1}{3}F\left(\frac{4M_{\mathrm{eff}}^2}{3m_{\mathrm{g}}^2}\right)\right)\right].
\end{aligned} \tag{3.5}$$

Therefore, those results demonstrate that in order to study the properties of the functions $J_{T,L}$ and $K_{T,L}$, one needs only to study the properties of the much simpler function $F(x)$. In fact, it is even possible to write the function $F(x)$ in closed form in terms of dilogarithms.[6] We are not going to make use of this possibility here since it is simpler to keep $F(x)$ in its integral form given by eq. (3.3).

One must stress the fact that all these functions depend only on the ratio of two masses. In the case of real photons ($Q^2 = 0$), we have in addition $M_{\rm eff}^2 = M_\infty^2$ and for $N_c = 3$ colors, we can write:

$$\frac{4M_\infty^2}{3m_{\rm g}^2} = \frac{8}{6+N_F}\,, \tag{3.7}$$

i.e. the temperature and strong coupling constant also drop out of this ratio. It appears that there is an additional (and purely accidental) simplification for $N_F = 2$ flavors: in this case, the differences $J_L - J_T$ and $K_L - K_T$ can be expressed in a very simple fashion:

$$\begin{aligned} J_L - J_T\Big|_{N_c=3,N_F=2} &= \pi\ln(2)\,, \\ K_L - K_T\Big|_{N_c=3,N_F=2} &= \frac{\pi}{4}(1-2\ln(2))\,. \end{aligned} \tag{3.8}$$

For the case of $N_F = 3$ flavors, the results are less simple, but one can still obtain explicit expressions:

$$\begin{aligned} J_L - J_T\Big|_{N_c=3,N_F=3} &= \pi\left[\frac{\pi^2}{8} - \frac{33}{8}\ln^2(2) + 3\ln(2)\ln(3) - \right. \\ &\qquad \left. -\frac{3}{2}\,{\rm Li}_2\left(\frac{3}{4}\right) - \frac{3}{2}\,{\rm Li}_2\left(-\frac{1}{2}\right)\right], \\ K_L - K_T\Big|_{N_c=3,N_F=3} &= \pi\left[\frac{1}{4} - \frac{\pi^2}{36} + \frac{\ln(2)}{8} - \frac{\ln(3)}{4} + \frac{11}{12}\ln^2(2) - \right. \\ &\qquad \left. -\frac{2}{3}\ln(2)\ln(3) + \frac{1}{3}\,{\rm Li}_2\left(\frac{3}{4}\right) + \frac{1}{3}\,{\rm Li}_2\left(-\frac{1}{2}\right)\right]. \end{aligned} \tag{3.9}$$

Had these exact formulas been known, the confusion due to the erroneous factor 4 in the numerical evaluation of these coefficients in [27] would have been avoided [40].

4. Asymptotic behavior of $J_{T,L}$ and $K_{T,L}$

4.1 Limit $M_{\rm eff} \ll m_{\rm g}$

Using the above results, we can recover in a rather simple and elegant way all the asymptotic limits given in [26] for $J_{T,L}$, as well as the limits used for $K_{T,L}$ in order to obtain the behavior

[6] Explicitly, we have:

$$F(x) = \frac{1}{4ip}\left[{\rm Li}_2\left(\frac{2p}{p+i}\right) - 2\,{\rm Li}_2\left(\frac{p}{p+i}\right) - {\rm Li}_2\left(\frac{2p}{p-i}\right) + 2\,{\rm Li}_2\left(\frac{p}{p-i}\right)\right.$$
$$\left. + 2\ln(2)\ln\left(\frac{i-p}{i+p}\right)\right], \qquad \text{with} \quad p \equiv \sqrt{x-1} \quad \text{and} \quad {\rm Li}_2(x) \equiv \sum_{n=1}^{+\infty}\frac{x^n}{n^2}\,. \tag{3.6}$$

of 2-loop dilepton production near the threshold $Q^2 = 4M_\infty^2$ (a region which is dominated by small values of $M_{\rm eff}^2$) [37].

To that effect, we need only the behavior of $F(x)$ when $x \to 0$. This is derived in the appendix A, where we prove that:

$$F(x) \underset{x\to 0^+}{=} \frac{1}{8}\ln^2\left(\frac{4}{x}\right) + \frac{\pi^2}{24} + \mathcal{O}\left(x\ln^2\left(\frac{1}{x}\right)\right). \tag{4.1}$$

Thanks to this formula, a trivial calculation gives[7]

$$\begin{aligned}
J_{\scriptscriptstyle T} &\underset{M_{\rm eff}\ll m_{\rm g}}{\approx} -\frac{\pi}{8}\ln^2\left(\frac{m_{\rm g}^2}{M_{\rm eff}^2}\right),\\
J_{\scriptscriptstyle L} &\underset{M_{\rm eff}\ll m_{\rm g}}{\approx} \frac{\pi\ln(3)}{4}\ln\left(\frac{m_{\rm g}^2}{M_{\rm eff}^2}\right),\\
K_{\scriptscriptstyle T} &\underset{M_{\rm eff}\ll m_{\rm g}}{\approx} \frac{\pi}{8}\left[\ln\left(\frac{m_{\rm g}^2}{M_{\rm eff}^2}\right) - 2\right],\\
K_{\scriptscriptstyle L} &\underset{M_{\rm eff}\ll m_{\rm g}}{\approx} -\frac{\pi\ln(3)}{8}.
\end{aligned} \tag{4.2}$$

These relations in fact go well beyond the results for $J_{\scriptscriptstyle T,L}$ obtained in [26], since in this new approach we obtain for free the prefactor of the leading term and we could even have calculated some subleading terms, down to the constant term.

4.2 Limit $M_{\rm eff} \gg m_{\rm g}$

Using eqs. (3.5), it is also very simple to obtain the behavior of the functions $J_{\scriptscriptstyle T,L}$ and $K_{\scriptscriptstyle T,L}$ in the opposite limit where $M_{\rm eff} \gg m_{\rm g}$. In order to do that, we need to know the behavior of $F(x)$ for large values of x. The following formula is also derived in appendix A:

$$F(x) \underset{x\gg 1}{=} \frac{\ln(x)}{2x} + \frac{1-\ln(2)}{x} + \frac{\ln(x)}{3x^2} + \frac{5-6\ln(2)}{9x^2} + \mathcal{O}\left(\frac{\ln(x)}{x^3}\right). \tag{4.3}$$

From this formula, it is easy to obtain

$$\begin{aligned}
J_{\scriptscriptstyle T} &\underset{M_{\rm eff}\gg m_{\rm g}}{\approx} -\frac{\pi}{8}\frac{m_{\rm g}^2}{M_{\rm eff}^2}\ln\left(\frac{M_{\rm eff}^2}{m_{\rm g}^2}\right),\\
J_{\scriptscriptstyle L} &\underset{M_{\rm eff}\gg m_{\rm g}}{\approx} \frac{\pi}{4}\frac{m_{\rm g}^2}{M_{\rm eff}^2}\ln\left(\frac{M_{\rm eff}^2}{m_{\rm g}^2}\right) \approx -2J_{\scriptscriptstyle T},\\
K_{\scriptscriptstyle T} &\underset{M_{\rm eff}\gg m_{\rm g}}{\approx} \frac{\pi}{48}\frac{m_{\rm g}^2}{M_{\rm eff}^2}\ln\left(\frac{M_{\rm eff}^2}{m_{\rm g}^2}\right),\\
K_{\scriptscriptstyle L} &\underset{M_{\rm eff}\gg m_{\rm g}}{\approx} -\frac{\pi}{24}\frac{m_{\rm g}^2}{M_{\rm eff}^2}\ln\left(\frac{M_{\rm eff}^2}{m_{\rm g}^2}\right) \approx -2K_{\scriptscriptstyle T}.
\end{aligned} \tag{4.4}$$

[7] These leading log formulas are not affected by the fact that the asymptotic expansion of $F(x)$ is slightly modified if x approaches 0 with negative values (see eq. (A.10)).

5. Beyond the HTL approximation

Recently, there has been some attempts to describe a quark-gluon plasma as a gas of quasi free quasiparticles. One such model [51] assumes in fact that these quasi-particles have the dispersion relations that one gets from the HTL approximation (and no width), and uses two free parameters in the temperature dependence of the coupling constant $g(T)$ (this dependence is motivated by the 1-loop running of the coupling constant, but there is some freedom in the choice of the renormalization point as a function of the temperature). This very simple model has been quite successful in reproducing all the available lattice results for the pressure of a QGP.

Moreover, one can calculate in this model the leading part of the self-energy of a space-like gluon (the model only makes an assumption for the time-like spectrum of excitations in the plasma – space-like properties like screening can then be calculated), and derive an expression for the Debye mass that also agrees qualitatively with the lattice results (this is now a prediction, and not a fit – the free parameters of the model were fitted so that the pressure is correctly reproduced). This is an interesting test because the HTL prediction for the Debye mass would indicate that it increases when one approaches the critical temperature T_c from above, while lattice results indicate that it becomes very small. Physically, this is explained as follows: when the temperature becomes close to T_c, the quasi-particles that populate the plasma become very massive [51, 52], which strongly suppresses the ability of a test charge to polarize the medium surrounding it; therefore, Debye screening becomes ineffective.

Note also that this simple quasi-particle model satisfies the conditions listed at the end of section 2. Let us use this sum-rule in order to formulate the photon rate of eq. (1.1) in a way that is suitable for its evaluation in such a quasi-particle model. It is easy to see that the result depends only on the four numbers $\mathrm{Re}\,\Pi_{_{T,L}}(\infty)$ and $\mathrm{Re}\,\Pi_{_{T,L}}(0)$. The first two are the plasmon mass (longitudinal) and the mass of the transverse gluon at zero momentum, and can be shown to be equal thanks to Slavnor-Taylor identities [48, 49, 50]. Physically, this property means that there is no way to distinguish transverse and longitudinal modes for a particle at rest. Therefore, we need only to introduce one plasmon mass:

$$\mathrm{Re}\,\Pi_{_T}(\infty) = \mathrm{Re}\,\Pi_{_L}(\infty) \equiv m_P^2\,. \tag{5.1}$$

The other numbers needed are squares of the screening masses for the transverse and longitudinal static gluon exchanges. The longitudinal screening mass is the familiar Debye mass:

$$m_D^2 \equiv \mathrm{Re}\,\Pi_{_L}(0)\,. \tag{5.2}$$

In the HTL approximation, there is no screening for the transverse static gluons, but this is not expected to hold generally. The corresponding screening mass is the magnetic mass, and is denoted

$$m_{\rm mag}^2 \equiv \mathrm{Re}\,\Pi_{_T}(0)\,. \tag{5.3}$$

At this stage, we keep this parameter only as a handle to grossly estimate the effect of having magnetic screening. However, one should have in mind that magnetic screening

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is not limited to a modification of $\Pi_T(0)$ and that the calculation becomes intrinsically non-perturbative when magnetic screening is important.

In terms of these parameters, it is straightforward to write down the expressions of $J_{T,L}$ and $K_{T,L}$ for photon production in a description where we use gluon propagators that are more general than the HTL propagators:

$$
\begin{aligned}
J_T &= \pi\left[F\left(\frac{4M_{\rm eff}^2}{m_{\rm mag}^2}\right) - F\left(\frac{4M_{\rm eff}^2}{m_P^2}\right)\right],\\
J_L &= \pi\left[F\left(\frac{4M_{\rm eff}^2}{m_D^2}\right) - F\left(\frac{4M_{\rm eff}^2}{m_P^2}\right)\right],\\
K_T &= \pi\left[-\frac{1}{8}\ln\left(\frac{m_{\rm mag}^2}{m_P^2}\right) + \frac{M_{\rm eff}^2}{m_P^2}F\left(\frac{4M_{\rm eff}^2}{m_P^2}\right) - \frac{M_{\rm eff}^2}{m_{\rm mag}^2}F\left(\frac{4M_{\rm eff}^2}{m_{\rm mag}^2}\right)\right],\\
K_L &= \pi\left[-\frac{1}{8}\ln\left(\frac{m_D^2}{m_P^2}\right) + \frac{M_{\rm eff}^2}{m_P^2}F\left(\frac{4M_{\rm eff}^2}{m_P^2}\right) - \frac{M_{\rm eff}^2}{m_D^2}F\left(\frac{4M_{\rm eff}^2}{m_D^2}\right)\right].
\end{aligned}
\tag{5.4}
$$

It is easy to check that these relations fall back to eqs. (3.5) if we set $m_{\rm mag}=0$, $m_P=m_{\rm g}$ and $m_D=\sqrt{3}m_{\rm g}$, which are the relations between masses in the HTL framework.

There is another general property of the processes of figure 2 which is worth mentioning here. Their rate in fact depends only on the combinations $J_T - J_L$ and $K_T - K_L$ (see eq. (1.1)) after one has summed the contributions of transverse and longitudinal gluons. Using the above formulas, we obtain:

$$
\begin{aligned}
J_T - J_L &= \pi\left[F\left(\frac{4M_{\rm eff}^2}{m_{\rm mag}^2}\right) - F\left(\frac{4M_{\rm eff}^2}{m_D^2}\right)\right],\\
K_T - K_L &= \pi\left[\frac{1}{8}\ln\left(\frac{m_D^2}{m_{\rm mag}^2}\right) + \frac{M_{\rm eff}^2}{m_D^2}F\left(\frac{4M_{\rm eff}^2}{m_D^2}\right) - \frac{M_{\rm eff}^2}{m_{\rm mag}^2}F\left(\frac{4M_{\rm eff}^2}{m_{\rm mag}^2}\right)\right].
\end{aligned}
\tag{5.5}
$$

In other words, all the dependence on the plasmon mass drops out for the processes of figure 2. This property is in fact reasonable since we are looking at processes that involve only space-like gluons, and it would have been surprising if the result had depended on the plasmon mass, a property of time-like gluons. Note also that these quantities remain finite even if the magnetic mass is very small or vanishing.

Therefore, assuming a vanishing magnetic mass, eq. (1.1) depends only on the ratio $M_{\rm eff}^2/m_D^2$, or more simply M_∞^2/m_D^2 for real photons. In the quasi-particle model of [51], M_∞ is determined by a fit of the lattice data for the pressure, and the Debye mass m_D can be calculated from there. This procedure leads to a ratio M_∞^2/m_D^2 that increases when the temperature approaches T_c from above, while it would be temperature-independent in the HTL framework. This fact alone would modify (reduce, in fact) the value of the numerical prefactor in the photon rate, compared to the value obtained in the HTL framework. It would be interesting to test what a model that reproduces successfuly the lattice pressure near T_c can say about a dynamical quantity like the photon rate. We expect that this model provides a better description of a QGP at temperatures that are not far above the critical temperature. Interestingly, it could soon become feasable to compare this idea with lattice evaluations of the photon rate since a lot of progress has been made in this area recently [52].

6. Resummation of ladder diagrams and LPM effect

6.1 Integral equation in momentum space

The authors of [32, 33] performed the resummation of all the ladder diagrams, as well as all the self-energy corrections that are required to preserve gauge invariance, in order to account for the LPM effect in the production of real photons. Indeed, such photons may have a formation time larger than the mean free path of the quarks in the medium [31], so that multiple quark scatterings contribute coherently to the formation of the photon.

In the formulation of [32, 33], the imaginary part of the retarded photon polarization tensor is given by[8]

$$\mathrm{Im}\,\Pi_{R\mu}^{\;\mu}(Q) \approx \frac{e^2 N_c}{8\pi}\int_{-\infty}^{+\infty} dp_0\,[n_F(r_0) - n_F(p_0)]\,\frac{p_0^2 + r_0^2}{p_0^2 r_0^2}\times \\ \times\,\mathrm{Re}\int \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\,\boldsymbol{p}_\perp\cdot\boldsymbol{f}(\boldsymbol{p}_\perp)\,, \tag{6.1}$$

where $r_0 \equiv p_0 + q_0$ and $\boldsymbol{f}(\boldsymbol{p}_\perp)$ is a dimensionless transverse vector that represents the resummed coupling of a quark to the transverse modes of the photon, satisfying the following integral equation

$$2\boldsymbol{p}_\perp = i\delta E\,\boldsymbol{f}(\boldsymbol{p}_\perp) + g^2 C_F T\int \frac{d^2\boldsymbol{l}_\perp}{(2\pi)^2}\mathcal{C}(\boldsymbol{l}_\perp)\,[\boldsymbol{f}(\boldsymbol{p}_\perp) - \boldsymbol{f}(\boldsymbol{p}_\perp + \boldsymbol{l}_\perp)]\,, \tag{6.2}$$

where $\delta E \equiv q_0(\boldsymbol{p}_\perp^2 + M_{\mathrm{eff}}^2)/(2p_0 r_0)$ and in which $\mathcal{C}(\boldsymbol{l}_\perp)$ is the collision integral defined by

$$\mathcal{C}(\boldsymbol{l}_\perp) \equiv \int \frac{dl_0 dl_z}{(2\pi)^2} 2\pi\delta(l_0 - l_z)\,\frac{1}{l_0}\times \\ \times\sum_{\alpha=L,T}\frac{2\,\mathrm{Im}\,\Pi_\alpha(L)}{(L^2 - \mathrm{Re}\,\Pi_\alpha(L))^2 + (\mathrm{Im}\,\Pi_\alpha(L))^2}\,P_\alpha^{\mu\nu}(L)\widehat{Q}_\mu\widehat{Q}_\nu\,, \tag{6.3}$$

with $\widehat{Q}_\mu \equiv (1, \boldsymbol{q}/q)$ and $P_{T,L}^{\mu\nu}(L)$ the transverse or longitudinal projector for a gluon of momentum L. Note that δE^{-1} is nothing but the typical formation time of the photon [31]. Note also that in an iterative solution of these integral equations, the first term that contributes to the imaginary part of the photon polarization tensor is of order g^2 since $\boldsymbol{f}(\boldsymbol{p}_\perp)$ is purely imaginary at order zero in the collision term. This reflects the fact that the direct production of a photon by the processes $q\bar{q} \to \gamma$ or $q \to q\gamma$ is kinematically forbidden. This remark ceases to be valid if δE can vanish, in which case an $i\epsilon$ prescription must be used, so that the zeroth order solution can have a real part. This happens if M_{eff}^2 can become negative, i.e. if $Q^2 > 4M_\infty^2$.

Using the fact that we have[9]

$$P_T^{\mu\nu}(L)\widehat{Q}_\mu\widehat{Q}_\nu = \frac{l_z^2}{l^2} - 1 = -P_L^{\mu\nu}(L)\widehat{Q}_\mu\widehat{Q}_\nu\,, \tag{6.6}$$

[8] Note that this equation includes only the two transverse modes of the photon, and is therefore incomplete for the production of virtual photons. It is however not difficult to include the longitudinal mode as well [53].

[9] One can use

$$P_T^{\mu\nu}(L) + P_L^{\mu\nu}(L) = g^{\mu\nu} - \frac{L^\mu L^\nu}{L^2}\,, \tag{6.4}$$

and introducing the variable $x \equiv l_0/l$, we can rewrite[10] the collision integral in eq. (6.3) as

$$\mathcal{C}(\boldsymbol{l}_\perp) = \frac{2}{\pi}\int_0^1 \frac{dx}{x}\left[\frac{\operatorname{Im}\Pi_L(x)}{(l_\perp^2+\operatorname{Re}\Pi_L(x))^2+(\operatorname{Im}\Pi_L(x))^2} - \right.$$
$$\left. - \frac{\operatorname{Im}\Pi_T(x)}{(l_\perp^2+\operatorname{Re}\Pi_T(x))^2+(\operatorname{Im}\Pi_T(x))^2}\right]. \tag{6.8}$$

At this point, the sum rule derived in section 2 gives directly the result of the integral over the variable x, so that the collision integral can be rewritten as:

$$\mathcal{C}(\boldsymbol{l}_\perp) = \frac{1}{l_\perp^2} - \frac{1}{l_\perp^2+3m_{\rm g}^2}. \tag{6.9}$$

We observe again that summing over the contributions of transverse and longitudinal gluon exchanges cancels the terms involving the plasmon mass.

6.2 Solution in the Bethe-Heitler regime

As a check, one can solve this integral equation iteratively up to the first order in the collision term. Indeed, the zeroth order term is purely imaginary, and drops out of the imaginary part of the photon polarization tensor (see eq. (6.1)). For the function $\boldsymbol{f}(\boldsymbol{p}_\perp)$, this expansion gives:

$$\operatorname{Re}\int\frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\boldsymbol{p}_\perp\cdot\boldsymbol{f}(\boldsymbol{p}_\perp) \underset{\mathcal{O}(\mathcal{C}^1)}{=} 8g^2C_FT\frac{(p_0r_0)^2}{q_0^2}\int\frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\frac{\boldsymbol{p}_\perp}{\boldsymbol{p}_\perp^2+M_{\rm eff}^2}\cdot$$
$$\cdot\int\frac{d^2\boldsymbol{l}_\perp}{(2\pi)^2}\mathcal{C}(\boldsymbol{l}_\perp)\left[\frac{\boldsymbol{p}_\perp}{\boldsymbol{p}_\perp^2+M_{\rm eff}^2} - \frac{\boldsymbol{p}_\perp+\boldsymbol{l}_\perp}{(\boldsymbol{p}_\perp+\boldsymbol{l}_\perp)^2+M_{\rm eff}^2}\right]. \tag{6.10}$$

At this stage, performing the angular integrations is elementary. Making use of the following identity:

$$\int_0^{+\infty} d(p_\perp^2)\left(\frac{1}{p_\perp^2+M_{\rm eff}^2} - \frac{1}{\sqrt{(p_\perp^2+l_\perp^2+M_{\rm eff}^2)^2-4p_\perp^2l_\perp^2}}\right) = 0, \tag{6.11}$$

this can be rewritten as:

$$\operatorname{Re}\int\frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\boldsymbol{p}_\perp\cdot\boldsymbol{f}(\boldsymbol{p}_\perp) \underset{\mathcal{O}(\mathcal{C}^1)}{=} -\frac{2g^2C_FT}{\pi^3}\frac{(p_0r_0)^2}{q_0^2}[J_T - J_L + 2K_T - 2K_L]. \tag{6.12}$$

---

and

$$\widehat{Q}_\mu\widehat{Q}_\nu\left(g^{\mu\nu} - \frac{L^\mu L^\nu}{L^2}\right) = 0 \quad \text{if} \quad l_0 = l_z \tag{6.5}$$

in order to obtain the second contraction.

[10]This change of coordinates has the following jacobian:

$$\left(1 - \frac{l_0^2}{l_0^2+l_\perp^2}\right)\frac{dl_0}{l_0}d(l_\perp^2) = \frac{dx}{x}d(l_\perp^2). \tag{6.7}$$

Finally, plugging eqs. (6.12) into eq. (6.1) gives

$$\mathrm{Im}\,\Pi_{R\,\mu}^{\ \mu}(Q) \underset{\mathcal{O}(\mathcal{C}^1)}{=} -\frac{e^2 g^2 N_c C_F}{2\pi^4}\frac{T}{q_0^2}\int_{-\infty}^{+\infty} dp_0 [n_F(r_0) - n_F(p_0)] \times \\ \times\ (p_0^2 + r_0^2)[J_T - J_L + 2K_T - 2K_L]\,, \qquad (6.13)$$

which is equivalent to eq. (1.1) for real photons ($Q^2 = 0$, $M_{\mathrm{eff}}^2 = M_\infty^2$). This proves the agreement between the perturbative approach followed in [37] and the resummation of the LPM corrections, if one formally keeps only the first order in the collision term. The fact that some terms proportional to Q^2 in eq. (1.1) are not recovered in this limit is due to the fact that the LPM resummation of [32, 33] is limited to the transverse modes of the produced photon, while massive photons also have a physical longitudinal mode.

6.3 Reformulation as a differential equation

Since the collision integral appearing under the integral over $\boldsymbol{l}_\perp$ is now known in closed form, it is possible to transform this integral equation into an ordinary differential equation by going to impact parameter space. We can first define

$$\boldsymbol{f}(\boldsymbol{p}_\perp) \equiv \int d^2\boldsymbol{b}\, e^{-i\boldsymbol{p}_\perp\cdot\boldsymbol{b}}\boldsymbol{g}(\boldsymbol{b})\,. \qquad (6.14)$$

Note that the order zero solution (in powers of the collision integral) of the integral equation is[11]

$$\boldsymbol{g}_0(\boldsymbol{b}) = -\frac{2}{\pi}\frac{p_0 r_0}{q_0}\nabla_b K_0(M_{\mathrm{eff}} b) = \frac{2}{\pi}\frac{p_0 r_0}{q_0} M_{\mathrm{eff}}\widehat{\boldsymbol{b}} K_1(M_{\mathrm{eff}} b)\,, \qquad (6.15)$$

where the K_i's are modified Bessel functions of the second kind. For the higher order terms $\boldsymbol{g}_1(\boldsymbol{b})$, one obtains the following equation for $\boldsymbol{g}_1$

$$i\frac{q_0}{2p_0 r_0}(M_{\mathrm{eff}}^2 - \Delta_\perp)\boldsymbol{g}_1(\boldsymbol{b}) + g^2 C_F T D(m_{\mathrm{g}} b)(\boldsymbol{g}_0(\boldsymbol{b}) + \boldsymbol{g}_1(\boldsymbol{b})) = 0\,, \qquad (6.16)$$

with

$$D(x) \equiv \frac{1}{2\pi}\left[\gamma + \ln\left(\frac{\sqrt{3}x}{2}\right) + K_0(\sqrt{3}x)\right]. \qquad (6.17)$$

In addition, we have also

$$\mathrm{Re}\int\frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\boldsymbol{p}_\perp\cdot\boldsymbol{f}(\boldsymbol{p}_\perp) = \lim_{b\to 0^+}\mathrm{Im}\,\nabla_\perp\cdot\boldsymbol{g}_1(\boldsymbol{b})\,. \qquad (6.18)$$

This differential equation can be further simplified by defining the following dimensionless quantities:

$$t \equiv \left|M_{\mathrm{eff}}^2\right| b^2\,,$$
$$u(t) \equiv \frac{\pi q_0}{2p_0 r_0}\boldsymbol{b}\cdot\boldsymbol{g}(\boldsymbol{b})\,. \qquad (6.19)$$

[11]This formula is valid for $M_{\mathrm{eff}}^2 > 0$. It remains valid for $M_{\mathrm{eff}}^2 < 0$, provided one sees then M_{eff} as a purely imaginary number. If one wants only functions of real arguments, M_{eff} must be replaced by its modulus, and the function K_1 becomes a Y_1.

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This transformation leads to

$$4tu_1''(t) - \mathrm{sign}(M_{\rm eff}^2)u_1(t) + 2ig^2C_{\!_F}\frac{p_0r_0}{q_0}\frac{T}{|M_{\rm eff}^2|}D\left(\frac{m_{\rm g}}{|M_{\rm eff}|}\sqrt{t}\right)(u_0(t)+u_1(t)) = 0\,, \quad (6.20)$$

where the prime denotes the differentiation with respect to t, and where[12]

$$u_0(t) \equiv \sqrt{t}K_1(\sqrt{t})\,. \quad (6.21)$$

The differential equation eq. (6.20) depends on two dimensionless quantities. One is the ratio $m_{\rm g}/M_{\rm eff}$ of two masses, while the prefactor $g^2C_{\!_F}Tp_0r_0/q_0M_{\rm eff}^2$ can be interpreted (up to logarithms) as the ratio of the photon formation time to the quark mean free path. It is therefore the average number of scatterings that can contribute coherently to the production of a photon. The relevant quantity for the photon production rate is then given by

$$\mathrm{Re}\int\frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\boldsymbol{p}_\perp\cdot\boldsymbol{f}(\boldsymbol{p}_\perp) = \frac{4p_0r_0}{\pi q_0}M_{\rm eff}^2\,\mathrm{Im}\,u_1'(0)\,. \quad (6.22)$$

This differential equation must be supplemented by boundary conditions in order to define uniquely the solution. First, we can see from the differential equation (6.16) that $\boldsymbol{g}_1(\boldsymbol{b})$ does not diverge at $b=0$ (the full $\boldsymbol{g}$ contains a divergence, which goes entirely in the zeroth-order term $\boldsymbol{g}_0$). This implies

$$u_1(0) = 0\,. \quad (6.23)$$

Unfortunately, we do not know the value of $u_1'(0)$ but instead we know the value of $u(\infty)$. Indeed, normalizability requires that $u(\infty)=0$ (another way to see it is to assume that $\boldsymbol{f}(\boldsymbol{p}_\perp)$ is regular enough, which implies that its Fourier transform is exponentially suppressed at large b – the regularity of $\boldsymbol{f}(\boldsymbol{p}_\perp)$ is equivalent to the integrability of $\boldsymbol{g}(\boldsymbol{b})$). Since we already have $u_0(\infty)=0$, this implies

$$u_1(\infty) = 0\,. \quad (6.24)$$

Therefore, the problem can be summarized as follows: we are looking for the (presumably complex) value of $u_1'(0)$ that takes us from $u_1(0)=0$ to $u_1(\infty)=0$. The imaginary part of this derivative term is then the coefficient that enters in the photon polarization tensor. Note also that since the solution space of this differential equation is a 2-dimensional complex linear space, having two complex boundary conditions is enough to uniquely determine a solution.

6.4 Numerical solution

6.4.1 General principle

This reformulation of eqs. (6.1) and (6.2) leads to a rather straightforward algorithm for a numerical solution. Differential problems with two-point boundary conditions are usually solved by iterative “shooting” methods [54], where one tries to make a guess for the missing initial condition (here, $u_1'(0)$) and then corrects this value based on the discrepancy between the resulting end-point and the expected one.

[12] If $M_{\rm eff}^2<0$, one has $u_0(t)\equiv i\sqrt{t}K_1(i\sqrt{t})$.

Things are in fact much simpler for an affine equation, since only two trials, plus the knowledge of a solution of the complete equation, are enough to find the value of $u_1'(0)$. Indeed, for a differential equation like (6.20), solutions have generically the following form:

$$u_1(t) = w(t) + \alpha_1 w_1(t) + \alpha_2 w_2(t)\,, \tag{6.25}$$

where $w(t)$ is a particular solution of the full equation, and $w_{1,2}(t)$ are two independent solutions of the corresponding homogeneous equation (i.e. (6.20) in which one would set u_0 to zero). Generically, it is convenient to choose these functions such that:

$$\begin{aligned} w(0) &= 0\,, & w'(0) &= 0\,, \\ w_1(0) &= 1\,, & w_1'(0) &= 0\,, \\ w_2(0) &= 0\,, & w_2'(0) &= 1\,. \end{aligned}$$

If these solutions are known (at least numerically), the condition $u_1(0) = 0$ implies

$$\alpha_1 = 0\,. \tag{6.26}$$

Then, using $u_1(+\infty) = 0$, the second coefficient α_2 is given by

$$\alpha_2 = \lim_{t\to+\infty} \frac{-w(t)}{w_2(t)}\,; \tag{6.27}$$

and thanks to our choice of initial conditions for w, w_1 and w_2, the value of $u_1'(0)$ that solves the problem is simply given by

$$u_1'(0) = \alpha_2\,. \tag{6.28}$$

6.4.2 Realistic algorithm

This algorithm is however not directly applicable in the case of eq. (6.20), because the point $t = 0$ is a singular point of the equation, and cannot be used to set initial conditions. Therefore, we have to chose another point in order to set the initial conditions. Let us call this point $t_0 > 0$, and assume

$$\begin{aligned} w(t_0) &= 0\,, & w'(t_0) &= 0\,, \\ w_1(t_0) &= 1\,, & w_1'(t_0) &= 0\,, \\ w_2(t_0) &= 0\,, & w_2'(t_0) &= 1\,, \end{aligned}$$

instead of eqs. (6.26). From these initial conditions, one must evolve numerically the functions w, w_1 and w_2 both forward and backward. Generically, the three functions diverge when $t \to +\infty$ (unless one has been very unlucky when choosing the initial conditions). The condition $u_1(+\infty) = 0$ then implies

$$0 = 1 + \alpha_1 r_1 + \alpha_2 r_2\,, \tag{6.29}$$

where $r_i \equiv \lim_{t\to+\infty} w_i(t)/w(t)$. This gives a first linear relation between the two coefficients α_1 and α_2. Then, from the condition $u_1(0) = 0$, one can obtain

$$\alpha_1 = \lim_{t\to0^+} -\frac{w_2(t) - r_2 w(t)}{r_1 w_2(t) - r_2 w_1(t)}\,. \tag{6.30}$$

At this point, the two coefficients $\alpha_{1,2}$ are known, and it is easy to find

$$u_1'(0) = \lim_{t\to 0^+} w'(t) + \alpha_1 w_1'(t) + \alpha_2 w_2'(t)\,. \tag{6.31}$$

This last step does not require any additional calculation, since the derivatives of $w, w_{1,2}$ are known numerically at this point. In summary, this algorithm reduces the problem of solving the integral equation (6.2) and then calculating the integral over $\boldsymbol{p}_\perp$ in eq. (6.1) to the numerical solution of a differential equation with three different initial conditions. A numerical analysis of eq. (6.1) based along these lines will be presented elsewhere [53].

7. Conclusions

In this paper, we have derived a simple sum rule that enables to perform analytically some of the integrals involved in the thermal 2-loop photon production rate. Several applications of this sum rule have been presented. This sum rule also plays a role in the calculation of the collision integral that appears in the resummation of the ladder diagrams involved in the calculation of the LPM effect.

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A. Asymptotic behavior of $F(x)$

A.1 Large x behavior of $F(x)$

At large values of the argument x, the asymptotic value of the function $F(x)$ introduced in eq. (3.3) is very easy to obtain:[13]

$$\begin{aligned}
F(x) &= \epsilon\int_0^1 du\frac{\tanh^{-1}(u)}{u^2+\epsilon} \qquad \text{with} \qquad \epsilon\equiv\frac{1}{x-1}\ll 1\\
&= \epsilon\int_0^1 du\frac{\tanh^{-1}(u)-u}{u^2+\epsilon} + \epsilon\int_0^1 du\frac{u}{u^2+\epsilon}\\
&= \epsilon\left[1-\ln(2)+\mathcal{O}\left(\epsilon\ln\left(\frac{1}{\epsilon}\right)\right)\right] + \epsilon\ln\left(\frac{1+\epsilon}{\epsilon}\right)\,,
\end{aligned} \tag{A.2}$$

[13]By subtracting more of the Taylor expansion of $\tanh^{-1}(u)$, one can go one order further in this asymptotic expansion, and obtain

$$F(x) \underset{x\to\infty}{\approx} \frac{\ln(x)}{2x} + \frac{1-\ln(2)}{x} + \frac{\ln(x)}{3x^2} + \frac{5-6\ln(2)}{9x^2} + \mathcal{O}\left(\frac{\ln(x)}{x^3}\right)\,. \tag{A.1}$$

i.e.

$$F(x) \underset{x\to+\infty}{=} \frac{\ln(x)}{2x} + \frac{1-\ln(2)}{x} + \mathcal{O}\left(\frac{\ln(x)}{x^2}\right) . \tag{A.3}$$

Let us add that if the variable x goes to $-\infty$, one can obtain the correct asymptotic behavior[14] by replacing x by $-x$ in the previous expression (in particular $\ln(x)$ by $\ln|x| - i\pi$) while dropping the imaginary part, so that the previous asymptotic formula simply becomes:

$$F(x) \underset{x\to-\infty}{=} \frac{\ln|x|}{2x} + \frac{1-\ln(2)}{x} + \mathcal{O}\left(\frac{\ln|x|}{x^2}\right) . \tag{A.4}$$

A.2 Small x behavior of $F(x)$

At small values of x, determining the expansion of $F(x)$ requires a little more work. Denoting $y \equiv 1/\sqrt{1-x} \approx 1 + x/2$, we have

$$\begin{aligned} F(x) &= \frac{y^2}{2}\int_0^1 du \ln\left(\frac{1+u}{1-u}\right)\frac{1}{y^2-u^2} \\ &= \frac{y}{4}\int_0^1 du \ln\left(\frac{1+u}{1-u}\right)\left[\frac{1}{y+u} + \frac{1}{y-u}\right] . \end{aligned} \tag{A.5}$$

Let us notice first that the term in $1/(y+u)$ is finite in the limit $y \to 1$. Therefore, since we do not want to go beyond the constant terms, we can simply replace y by 1 in this term and get

$$\frac{1}{4}\int_0^1 du \ln\left(\frac{1+u}{1-u}\right)\frac{1}{1+u} = -\frac{1}{4}\int_0^1 dv \frac{\ln(v)}{1+v} = \frac{\pi^2}{48} . \tag{A.6}$$

The other term can be separated in two parts, the first one being

$$\begin{aligned} \frac{y}{4}\int_0^1 du \frac{\ln(1+u)}{y-u} &= \frac{y}{4}\int_0^1 \frac{\ln(1+u)-\ln(2)}{y-u} + \frac{y}{4}\int_0^1 \frac{\ln(2)}{y-u} \\ &\approx \frac{1}{4}\int_0^1 \frac{dv}{v}\ln\left(\frac{1-v}{2}\right) + \frac{\ln(2)}{4}\ln\left(\frac{2}{x}\right) \\ &\approx -\int_0^{\frac{1}{2}} \frac{dv}{4}\frac{\ln(1-v)}{1-v} + \int_0^{\frac{1}{2}} \frac{dv}{4}\frac{\ln(1-v)}{v(1-v)} + \frac{\ln(2)}{4}\ln\left(\frac{2}{x}\right) \\ &\approx \frac{\ln^2(2)}{8} - \frac{\pi^2}{48} + \frac{\ln(2)}{4}\ln\left(\frac{2}{x}\right) , \end{aligned} \tag{A.7}$$

up to terms that vanish when $x \to 0^+$. Seemingly, the second piece is:

$$\begin{aligned} -\frac{y}{4}\int_0^1 du \frac{\ln(1-u)}{y-u} &= \frac{y}{4}\int_0^1 du - \frac{\ln(y-u) + \ln\left(1 - \frac{y-1}{y-u}\right)}{y-u} \\ &\approx \frac{1}{8}\ln^2\left(\frac{2}{x}\right) + \frac{1}{4}\sum_{n=1}^{+\infty}\frac{(y-1)^p}{p}\int_0^1 \frac{du}{(y-u)^{p+1}} \\ &\approx \frac{1}{8}\ln^2\left(\frac{2}{x}\right) + \frac{\pi^2}{24} . \end{aligned} \tag{A.8}$$

[14]If x is negative, $F(x)$ should be understood with a principal value prescription (see [37] for more details).

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Collecting all the bits and pieces, we finally obtain:

$$F(x) \underset{x\to 0+}{=} \frac{1}{8}\ln^2\left(\frac{4}{x}\right) + \frac{\pi^2}{24} + \mathcal{O}\left(x\ln^2\left(\frac{1}{x}\right)\right). \tag{A.9}$$

When x approaches 0 by negative values, it is sufficient to replace x by $-x$ in the previous formula (i.e. $\ln(x)$ by $\ln|x| - i\pi$) and retain only the real part. Since the logarithm is squared, the $-i\pi$ modifies the constant term as follows:

$$F(x) \underset{x\to 0-}{=} \frac{1}{8}\ln^2\left|\frac{4}{x}\right| - \frac{\pi^2}{12} + \mathcal{O}\left(x\ln^2\left(\frac{1}{x}\right)\right). \tag{A.10}$$

B. For real photons: $J_T = -J_L$ if $M_\infty = m_{\rm g}$

In the production of real photons, one can notice numerically that the coefficients J_T and J_L are equal if the gluon plasmon mass $m_{\rm g}$ is equal to the quark asymptotic mass M_∞ [41] (for real photons $M_{\rm eff} = M_\infty$). We present here an analytic proof of this statement. When $M_\infty = m_{\rm g}$ and $Q^2 = 0$, we have:

$$\begin{aligned} J_L + J_T &= \pi\left[F\left(\frac{4}{3}\right) - 2F(4)\right] \\ &= \pi\left[3\int_0^1 du \frac{\tanh^{-1}(u)}{u^2+3} - 2\int_0^1 du \frac{\tanh^{-1}(u)}{3u^2+1}\right]. \end{aligned} \tag{B.1}$$

Noticing that the function

$$G(x) \equiv \int_0^1 du \frac{\tanh^{-1}(u)}{u^2+x^2} \tag{B.2}$$

obeys the differential equation

$$G(x) + xG'(x) = \frac{1}{2}\frac{\ln\left(\frac{1+x^2}{4x^2}\right)}{1+x^2}, \tag{B.3}$$

and solving it, one can prove the following formula

$$\begin{aligned} x\int_0^1 du \frac{\tanh^{-1}(u)}{u^2+x^2} &= \ln(2)\tan^{-1}\left(\frac{1}{x}\right) - \frac{1}{2}\int_0^{\frac{1}{x}} du \frac{\ln(1+u^2)}{1+u^2} \\ &= \ln(2)\tan^{-1}\left(\frac{1}{x}\right) + \int_0^{\tan^{-1}(1/x)} d\theta \ln(\cos(\theta)). \end{aligned} \tag{B.4}$$

Using this intermediate result, it is easy to rewrite $J_T + J_L$ as

$$J_T + J_L = \frac{\pi}{\sqrt{3}}\left[3\int_0^{\frac{\pi}{6}} d\theta \ln(\cos(\theta)) - 2\int_0^{\frac{\pi}{3}} d\theta \ln(\cos(\theta)) - \frac{\pi\ln(2)}{6}\right]. \tag{B.5}$$

Making then use of the following relations

$$
\begin{aligned}
\int_0^{\frac{\pi}{2}} d\theta \ln(\cos(\theta)) &= -\frac{\pi \ln(2)}{2}\,,\\
\int_0^{\frac{\pi}{3}} d\theta \ln(\cos(\theta)) &= -\frac{\pi \ln(2)}{2} - \int_0^{\frac{\pi}{6}} d\theta \ln(\sin(\theta))\,,\\
\int_0^{\frac{\pi}{6}} d\theta \ln(\cos(\theta)) &= -\frac{\pi \ln(2)}{2} - \int_0^{\frac{\pi}{3}} d\theta \ln(\sin(\theta))\,,\\
\int_0^{\frac{\pi}{6}} d\theta \ln(\sin(\theta)) + \int_0^{\frac{\pi}{6}} d\theta \ln(\cos(\theta)) &= \frac{1}{2}\int_0^{\frac{\pi}{3}} d\theta \ln(\sin(\theta)) - \frac{\pi \ln(2)}{6}\,,
\end{aligned}
\tag{B.6}
$$

it is straightforward to check that

$$
J_T + J_L = 0 \tag{B.7}
$$

when $M_\infty = m_{\rm g}$. This is an interesting non trivial (and exact) identity which cannot be obtained by any simple other method, mainly because there is no useful asymptotic formula near the point $M_\infty = m_{\rm g}$. Even if this identity is purely anecdotical, its derivation illustrates the power of the sum rule obtained in this paper.

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